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AUTOCONVOLUTION EQUATIONS OF THE THIRD KIND WITH ABEL INTEGRAL

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ABSTRACT. In this paper a class of autoconvolution equations of the third kind with additional fractional integral is investigated. Two general existence theorems are proved, and a new type of solutions is shown for an exceptional equation of this class.

1. Introduction. In the paper [**2**] Berg and the author firstly investigate the general autoconvolution equation of the third kind with coefficient $k(x) \sim Ax$ as $x \to 0$ and without a free term. These investigations are being continued in recent papers by the author jointly with Hofmann and Janno [**5, 7**–**11**]. In particular, in [**9**] the case $k(x) \sim Ax^\alpha$, $\alpha > 0$ and in [**11**] the cases $k(x) \sim Ax$ and $k(x) \sim Ax^{1/2}$ with a free term $p(x) \sim -\gamma^2$,$\gamma > 0$ as $x \to 0$ have been dealt with.

In the present paper the more general equation with an additional fractional integral

(1.1)
$$k(x)y(x) = \int_0^x y(\xi)y(x-\xi)\,d\xi + \frac{\nu}{B(\alpha,\alpha)}\int_0^x y(\xi)(x-\xi)^{\alpha-1}d\xi + p(x)$$

where $\nu \in \mathbf{R}$, $k(x) \sim Ax^\alpha$ ($\alpha > 0$) and $p(x) \sim -\gamma^2 x^{2\alpha-1}$ ($\gamma > 0$) or $p(x) = o(x^{2\alpha-1})$ as $x \to 0$ is treated. For $\nu = 0$ with $\gamma = 0$ this equation has been considered in [**9**], for $\nu = 0$, $\alpha = 1/2$ with $\gamma > 0$ in [**11**]. Again using Janno's theorem [**6**] in the iteration method with weighted norms in C space, we prove the existence of a one-parametric family of (real) solutions and an additional solitary solution in the main case $\gamma > 0$. These solutions also hold for $\gamma = 0$ with the exception of

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