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INEQUALITIES OF THE MARKOV TYPE FOR PARTIAL DERIVATIVES OF POLYNOMIALS IN SEVERAL VARIABLES

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1. Introduction and main results. Markov-type inequalities give upper bounds for the derivatives of an algebraic polynomial by the polynomial itself. To be more precise, they provide a constant C such that $\|D^\nu f\| \le C\|f\|$ for all polynomials of degree at most n, where D is the operator of differentiation. The constant C depends on n, on the order ν of the derivative, and on the norm $\|\cdot\|$. We here consider the case where $\|\cdot\|$ is one of the classical L^2 norms and study the problem of extending such inequalities to the situation when f is a polynomial of several variables and D^ν is replaced by a partial differential operator.

Let $\mathcal{P}_n$ be the linear space of all polynomials $f(t) = \sum_{j=0}^n f_j t^j$ of degree at most n with complex coefficients f_j. We equip $\mathcal{P}_n$ with one of the classical Hermite, Laguerre, or Gegenbauer norms. These are defined by

$$\|f\|^2 = \int_{-\infty}^{\infty} |f(t)|^2 e^{-t^2}\, dt, \tag{1}$$

$$\|f\|^2 = \int_0^{\infty} |f(t)|^2 t^\alpha e^{-t}\, dt, \tag{2}$$

$$\|f\|^2 = \int_{-1}^{1} |f(t)|^2 (1-t^2)^\alpha\, dt, \tag{3}$$

where $\alpha > -1$ is a parameter. Given a polynomial

$$p(\xi) = \xi^m + p_{m-1}\xi^{m-1} + \cdots + p_0,$$

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