SOLVING EXTERIOR NEUMANN BOUNDARY VALUE PROBLEMS FOR BELTRAMI FIELDS THROUGH THE BELTRAMI SYSTEM

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ABSTRACT. The Neumann boundary value problem for Beltrami fields $\nabla \times u = ku$ with constant k is studied in exterior domains. The problem is approached by considering the extended Beltrami system for which the unique solvability of the Dirichlet problem and of the Neumann problem are shown with a boundary integral equation method. The boundary integral equations for the Neumann problem of the Beltrami fields and their solvability follow from the Beltrami system results.

1. Introduction. A vector field $u : \mathbf{R}^3 \to \mathbf{C}^3$ is a Beltrami field if

$$\nabla \times u = \lambda u, \tag{1}$$

where the proportionality factor λ, in general, is a space dependent scalar function. Beltrami fields appear in plasma physics, electromagnetics and fluid mechanics [**2, 9, 16, 18, 19, 24, 29, 31**]. In some sense, Beltrami fields are in between the scalar acoustic fields and the vector valued electromagnetic fields. In this paper, we study the right-handed Beltrami fields with constant $\lambda = k > 0$. In this case, the Beltrami fields satisfy the Helmholtz equation with the wave number k. The left-handed case, $k < 0$, is similar.

The Neumann boundary value of a Beltrami field is the normal component of the field, which corresponds to the flow through the boundary. The solvability of the interior Neumann boundary value problem for Beltrami fields has been studied and solved in [**1, 3, 10, 12, 13, 14, 21, 30**], also with nonconstant proportionality factors