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A VARIATIONAL APPROACH FOR A NONLOCAL AND NONVARIATIONAL ELLIPTIC PROBLEM

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ABSTRACT. We prove results concerning the existence of solutions for the problem

$$-a(x, \int_\Omega u\,dx)\Delta u = f(x,u) \quad \text{in } \Omega,$$

where Ω is a bounded regular domain and $f : \overline{\Omega} \times \mathbf{R} \to \mathbf{R}$ is a function having subcritical growth. Although we are facing a problem with lack of variational structure we will be able to apply variational technique (the Mountain pass theorem) by suitably using a device introduced in [**6**].

1. Introduction. In this paper we investigate questions of existence of solutions for the following problem

$$(P_1) \qquad \begin{cases} -a\left(x, \int_\Omega u\,dx\right)\ \Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \\ u > 0 & \text{for all } x \in \Omega, \end{cases}$$

where $\Omega \subset \mathbf{R}^N$ is a bounded smooth domain, $N \geq 3$ and the functions a and f enjoy the following assumptions:

The function $a : \overline{\Omega} \times \mathbf{R} \to \mathbf{R}$ is continuous, and there are constants a_0, a_∞, R_1 and L_1 such that

$$(a_1) \qquad 0 < a_0 \leq a(x,t) \leq a_\infty \quad \text{for all } (x,t) \in \overline{\Omega} \times \mathbf{R},$$

and

$$(a_2) \qquad |a(x,s_1) - a(x,s_2)| \leq L_1 |s_1 - s_2|,$$

for all $s_1, s_2 \in [0, R_1]$ and for all $x \in \overline{\Omega}$.

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