JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 22, Number 3, Fall 2010

ROBUST GCV CHOICE OF THE REGULARIZATION PARAMETER FOR CORRELATED DATA

MARK A. LUKAS

Communicated by Kendall Atkinson

Dedicated to Charles W. Groetsch

ABSTRACT. We consider Tikhonov regularization of linear inverse problems with discrete noisy data containing correlated errors. Generalized cross-validation (GCV) is a prominent parameter choice method, but it is known to perform poorly if the sample size n is small or if the errors are correlated, sometimes giving the extreme value 0. We explain why this can occur and show that the robust GCV methods perform better. In particular, it is shown that, for any data set, there is a value of the robustness parameter below which the strong robust GCV method ($\mathrm{R_1GCV}$) will not choose the value 0. We also show that, if the errors are correlated with a certain covariance model, then, for a range of values of the unknown correlation parameter, the "expected" $\mathrm{R_1GCV}$ estimate has a near optimal rate as $n \to \infty$. Numerical results for the problem of second derivative estimation are consistent with the theoretical results and show that $\mathrm{R_1GCV}$ gives reliable and accurate estimates.

1. Introduction. Consider the problem of estimating a function or vector f_0 from discrete noisy data $y_i = L_i f_0 + \varepsilon_i$, $i = 1, \dots, n$, where L_i are linear functionals and ε_i are errors. In particular, we consider a linear ill-posed operator equation $Kf(x) = g(x)$, e.g., a first kind Fredholm integral equation, where the functionals are $L_i f = Kf(x_i)$, $i = 1, \dots, n$. Another special case is the data smoothing problem, where $L_i f = f(x_i)$. The general problem also includes a discretized operator equation or other finite dimensional linear model, in which case $L_i \mathbf{f} = K\mathbf{f}_i$, where $\mathbf{f} \in \mathbf{R}^q$, $q \leq n$, and K is the $n \times q$ model or design matrix.

2010 AMS *Mathematics subject classification.* Primary 65J20, 65J22, 45Q05, 62G08.

Received by the editors on October 4, 2009, and in revised form on March 14, 2010.

DOI:10.1216/JIE-2010-22-3-519