JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 22, Number 3, Fall 2010

AN ITERATIVE METHOD FOR TIKHONOV REGULARIZATION WITH A GENERAL LINEAR REGULARIZATION OPERATOR

MICHIEL E. HOCHSTENBACH AND LOTHAR REICHEL

Communicated by Kendall Atkinson

Dedicated to Charles W. Groetsch

ABSTRACT. Tikhonov regularization is one of the most popular approaches to solve discrete ill-posed problems with error-contaminated data. A regularization operator and a suitable value of a regularization parameter have to be chosen. This paper describes an iterative method, based on Golub-Kahan bidiagonalization, for solving large-scale Tikhonov minimization problems with a linear regularization operator of general form. The regularization parameter is determined by the discrepancy principle. Computed examples illustrate the performance of the method.

1. Introduction. We are concerned with the solution of large minimization problems

$$\min_{x\in\mathbf{C}^n} \|Ax-b\|, \qquad A\in\mathbf{C}^{m\times n}, \qquad b\in\mathbf{C}^m, \tag{1.1}$$

where $\|\cdot\|$ denotes the Euclidean vector norm and the matrix A is assumed to have many singular values of different orders of magnitude close to the origin. In particular, the ratio between the largest and smallest singular values is very large and therefore the solution of (1.1) is very sensitive to perturbations in the vector b. Minimization problems with matrices of this kind arise, for instance, from the discretization of ill-posed problems, such as Fredholm integral equations of the first kind. They are commonly referred to as discrete ill-posed

---

2010 AMS *Mathematics subject classification.* Primary 65R30, 65R32, 65F10, 65F22.
Keywords and phrases. Discrete ill-posed problem, iterative method, Tikhonov regularization, general linear regularization operator, discrepancy principle.
Received by the editors on February 9, 2010, and in revised form on June 21, 2010.

DOI:10.1216/JIE-2010-22-3-465