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LOCAL REGULARIZATION OF NONLINEAR VOLTERRA EQUATIONS OF HAMMERSTEIN TYPE

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ABSTRACT. The method of local regularization has been shown to be an effective tool for the reconstruction of solutions of linear and nonlinear inverse problems, especially those problems with special structure or for which non-smooth solutions are expected. In the case of Volterra problems, the method retains the causal structure of the original problem, in contrast to classical regularization methods, and leads to very fast sequential numerical algorithms to solve the inverse problem. Local regularization can be viewed as a generalization of *simplified* (or Lavrentiev) regularization studied by Groetsch and others, and as such can be applied to a wider variety of inverse problems; however, local regularization does not require an a priori estimate of the solution's initial value and, even if this value is known, in numerical tests local regularization frequently outperforms simplified regularization in the quality of reconstructed solution.

In this paper, we study the application of local regularization to the nonlinear Volterra problem of Hammerstein type. We improve upon the results of Lamm and Dai [**25**], where the localized approach led to a two-step solution method, i.e., one regularized linear step followed by one fully nonlinear step. Here we instead take advantage of the local nature of the method in order to simultaneously implement regularization while providing for an effective linearization strategy. The resulting method requires solving a nonlinear equation at one point only, for the initial value of the unknown solution. Thereafter the solution is reconstructed in a fast, sequential, and fully linear manner.

We present convergence results for this new method, discuss its numerical implementation and illustrate its use with numerical examples in which we compare the results of local regularization with another method well-suited for Volterra problems, the method of simplified (or Lavrentiev) regularization. In addition, we show how a modified discrepancy

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