AN EXTENSION OF THE
VARIATIONAL INEQUALITY APPROACH
FOR OBTAINING CONVERGENCE RATES IN
REGULARIZATION OF NONLINEAR ILL-POSED PROBLEMS

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ABSTRACT. Convergence rates results for Tikhonov regularization of nonlinear ill-posed operator equations in abstract function spaces require the handling of both smoothness conditions imposed on the solution and structural conditions expressing the character of nonlinearity. Recently, the distinguished role of variational inequalities holding on some level sets was outlined for obtaining convergence rates results. When lower rates are expected such inequalities combine the smoothness properties of solutions and forward operators in a sophisticated manner. In this paper, using a Banach space setting we are going to extend the variational inequality approach from Hölder rates to more general rates including the case of logarithmic convergence rates.

1. Introduction. With the monograph [**8**] Charles Groetsch presented an extremely well-readable introduction to the theory of Tikhonov regularization of ill-posed operator equations in Hilbert spaces. For linear ill-posed problems in that book the ingredients and conditions for obtaining convergence rates, the role of source conditions and the phenomenon of saturation are outlined. The ill-posedness of a linear operator equation describing an inverse problem with 'smoothing' forward operator in Hilbert spaces corresponds with the fact that