MODIFIED TIKHONOV REGULARIZATION FOR NONLINEAR ILL-POSED PROBLEMS IN BANACH SPACES

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ABSTRACT. We present a variant of Tikhonov regularization for nonlinear ill-posed problems in Banach spaces, where the convergence rate $O(\delta)$ for the Bregman distance is obtained under the same conditions as this rate is achieved for standard Tikhonov regularization. However, in this variant the regularization parameter can be chosen a-priori and independently from the condition on the exact solution.

1. Introduction. We consider nonlinear ill-posed problems

$$(1.1) \qquad\qquad F(x) = y,$$

where $F : \mathcal{D}F \subset X \to Y$ is a nonlinear bounded operator between Banach spaces. In practice only noisy data y^δ are available, where δ denotes the noise level. Throughout this paper we will assume that $\|y - y^\delta\| \leq \delta$.

Due to the ill-posedness, one has to use regularization methods to obtain stable approximations for an exact solution $x^\dagger$ of problem (1.1). A widely used method is Tikhonov regularization, where the regularized solution, x_α^δ, is a minimizer of the functional

$$(1.2) \qquad\qquad \frac{1}{p}\|F(x) - y^\delta\|^p + \alpha R(x), \quad \alpha > 0,$$

where $R(x)$ is a penalty term.

This method is well understood if F is an operator between Hilbert spaces, $p = 2$ and $R(x) = \|x - x_*\|^2$ (see, e.g., [3]). It turns out that in several situations Tikhonov regularization in Hilbert spaces does not yield good results, since it has the tendency to smooth the

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