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A CONVERGENCE ANALYSIS OF THE MIDPOINT RULE FOR FIRST KIND VOLTERRA INTEGRAL EQUATIONS WITH NOISY DATA

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ABSTRACT. In this paper we study convergence of the midpoint rule for linear Volterra integral equations of the first kind as a regularization method. We prove convergence and convergence rates as the noise level tends to zero. Numerical tests for an application to a thermoacoustic inverse problem illustrate performance of the method.

1. Introduction. In this paper we consider linear Volterra integral equations of the first kind

$$\int_a^x \mathbf{k}(x,\xi) q(\xi)\, d\xi = f(x) \quad x \in (a,b), \tag{1}$$

and their regularization by application of the midpoint quadrature rule to discretize the integral in (1). Regularization is necessary due to the fact that for a smooth kernel $\mathbf{k}$, (1) is ill-posed in the sense that its solution q (as an $L^{p/(p-1)}$ function) does not depend continuously on the data f (as an L^p function). Usually the data is not given exactly but only a noisy version f^δ is available, so that the lack of stability becomes crucial. We will here assume that we have an estimate δ of the noise level with respect to the L^∞ norm

$$\|f - f^\delta\|_{L^\infty} \le \delta. \tag{2}$$

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