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ERROR BOUNDS FOR SPECTRAL ENHANCEMENT WHICH ARE BASED ON VARIABLE HILBERT SCALE INEQUALITIES

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ABSTRACT. Spectral enhancement–which aims to undo spectral broadening–leads to integral equations which are ill-posed and require special regularization techniques for their solution. Even when an optimal regularization technique is used, however, the errors in the solution, which originate in data approximation errors, can be substantial and it is important to have good bounds on these errors in order to select appropriate enhancement methods. A discussion of the causes and nature of broadening provides regularity or source conditions which are required to obtain bounds for the regularized solution of the spectral enhancement problem. Only in special cases do the source conditions satisfy the requirements of the standard convergence theory for ill-posed problems. Instead we have to use variable Hilbert scales and their interpolation inequalities to get error bounds. The error bounds in this case turn out to be of the form $O(\varepsilon^{1-\eta(\varepsilon)})$ where ε is the data error and $\eta(\varepsilon)$ is a function which tends to zero when ε tends to zero. The approach is demonstrated with the Eddington correction formula and applied to a new spectral reconstruction technique for Voigt spectra. In this case $\eta(\varepsilon) = O(1/\sqrt{|\log \varepsilon|})$ is found.

1. Introduction. One of the computational challenges in spectroscopy is the separation of overlapping spectral lines. This separation can be achieved by computationally narrowing the spectral lines and thus enhancing the resolution or correcting the spectrum. The class of methods of resolution enhancement considered here is based on the solution of linear Fredholm integral equations of the first kind using observed data for the right hand side. The basic approach was first analyzed in [**2**] but it goes back in principle to work by Stokes [**47**]. The effect of data errors has to be analyzed carefully, especially since

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