JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 22, Number 2, Summer 2010

THE REGULARIZING LEVENBERG-MARQUARDT SCHEME IS OF OPTIMAL ORDER

MARTIN HANKE

Communicated by Kendall Atkinson

ABSTRACT. We prove that the regularizing Levenberg-Marquardt scheme, introduced for nonlinear ill-posed problems, achieves order optimal accuracy under standard assumptions on the nonlinearity of the underlying operator equation.

1. Introduction. The Levenberg-Marquardt method is a Newton-type method for nonlinear least-squares problems that is treated in many numerical optimization textbooks, cf., e.g., Kelley [**14**]. In each iteration of the Levenberg-Marquardt method the nonlinear operator is linearized around the current approximation, and the original problem is turned into a linear least squares problem with a quadratic inequality constraint. This constraint is derived on the grounds that one can only trust in the linearization within a certain neighborhood of the present approximation (*trust region*). Eventually, this leads to the same linear equation to be solved as in Tikhonov regularization, the corresponding regularization parameter being coupled with the Lagrange parameter associated with the constrained problem.

The Levenberg-Marquardt method is often used as a black box method for parameter identification problems, regardless of whether these are well-posed or not. Its convergence analysis, however, relies on the assumption that the derivative of the nonlinear operator is continuously invertible near the exact solution, cf. [**14**], which irrevocably fails to hold for ill-posed problems. To adjust the method to ill-posed problems we therefore proposed a modification of the Levenberg-Marquardt approach back in 1997 [**6**], to be called the *regularizing Levenberg-Marquardt scheme* below, which uses a different quadratic constraint that assesses the reliability of the right-hand side of the linearized problem rather than the trust region of the linearization. With

2010 AMS *Mathematics subject classification.* Primary 65J15, 65J20.
Keywords and phrases. Levenberg-Marquardt method, nonlinear ill-posed problems, regularization method, discrepancy principle, order optimal convergence rates.
Received by the editors on July 28, 2009.

DOI:10.1216/JIE-2010-22-2-259