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REGULARIZATION OF ILL-POSED LINEAR EQUATIONS BY THE NON-STATIONARY AUGMENTED LAGRANGIAN METHOD

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ABSTRACT. In this paper, we make a convergence rates analysis of the non-stationary Augmented Lagrangian Method for the solution of linear inverse problems. The motivation for the analysis is the fact that the Tikhonov-Morozov method is a special instance of the Augmented Lagrangian Method. In turn, the latter is also equivalent to iterative Bregman distance regularization, which received much attention in the imaging literature recently.

We base the analysis of the Augmented Lagrangian Method on convex duality arguments. Thereby, we can reprove some of the convergence (rates) results for the Tikhonov-Morozov Method. In addition, by the novel analysis we can prove properties of the dual variables of the Augmented Lagrangian methods. Reinterpretation of the dual variables for the Tikhonov-Morozov method gives some new convergence rate results for the linear functionals of the regularized solutions. As a benchmark for achievable convergence rates of the Augmented Lagrangian Method in the general convex context we use the results on evaluation of unbounded operators of Groetsch [**14**], which is a special instance of the Tikhonov-Morozov method. In addition we derive the flow, which interpolates the iterates of the Augmented Lagrangian Method and shows the relation to Showalter's method.

1. Introduction. In this paper, we are concerned with solving constrained optimization problems

$$J(u) \longrightarrow \min \quad \text{subject to } Ku = g, \tag{1}$$

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