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EXISTENCE AND APPROXIMATIONS OF SOLUTIONS TO SOME FRACTIONAL ORDER FUNCTIONAL INTEGRAL EQUATIONS

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ABSTRACT. In this paper we shall study a fractional order functional integral equation. In the first part of the paper, we proved the existence and uniqueness of mild and global solutions in a Banach space. In the second part of the paper, we used the analytic semigroups theory of linear operators and the fixed point method to establish the existence, uniqueness and convergence of approximate solutions of the given problem in a separable Hilbert space. We also proved the existence and convergence of Faedo-Galerkin approximate solution to the given problem. Finally, we give an example.

1. Introduction. We consider the following fractional order evolution equation in a Banach space $(X, \|.\|)$:

$$\begin{aligned} u(t) = u_0 &+ \frac{1}{\Gamma(\beta)} \int_0^t (t-\theta)^{\beta-1}(-Au(\theta))\, d\theta \\ &+ \frac{1}{\Gamma(\beta)} \int_0^t (t-\theta)^{\beta-1} f(\theta, u(\theta), u(a(\theta)))\, d\theta, \end{aligned} \tag{1.1}$$

where A is a closed linear operator defined on a dense set and $0 < \beta < 1$, $0 < T < \infty$. We assume that $-A$ is the infinitesimal generator of an analytic semigroup $\{S(t) : t \geq 0\}$ in X, Γ is the gamma function and $u(0) = u_0 \in X$. The functions f and a satisfy certain conditions to be specified later.

Regarding earlier works on existence and uniqueness of different type of solutions to fractional differential equations we refer to [**1, 8–14, 24**] and references cited in these papers.

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