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NORM ESTIMATES FOR A PARTICULAR WEIGHTED INTEGRAL OPERATOR

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ABSTRACT. We provide norm estimates for a particular integral operator on Hardy and Bergman spaces of analytic functions on the unit disc.

1. Introduction. Let λ_i, $x_i \in [-1,1]$, $i = 1,2$, be such that $|x_i \pm \lambda_i| \le 1$, $i = 1,2$. The linear segment $S_z = [x_1+\lambda_1 z, x_2+\lambda_2 z]$ is a subset of $\mathbf{D}$ for any $z \in \mathbf{D}$. We consider the function $r_z(t) = [S_z]t + (x_1+\lambda_1 z)$, $0 < t < 1$, $z \in \mathbf{D}$, where we define $[S_z] = (x_2+\lambda_2 z) - (x_1+\lambda_1 z)$. Let A, B, be linear complex functions with real coefficients. We assume that $[A(r_z(t))z + B(r_z(t))]^{-1}$ is bounded as a complex function of the variable t, for any $z \in \mathbf{D}$.

In this article we consider the integral operator

$$I(f)(z) = \frac{1}{[S_z]} \int_{S_z} \frac{f(\zeta)}{A(\zeta)z + B(\zeta)}\, d\zeta, \tag{1}$$

where f is an analytic function on the unit disc. For particular choices of the linear segment S_z and the functions A, B, we obtain certain well-known operators, like the Cesàro integral operator ([**6, 10, 11, 12**])

$$\mathcal{C}(f)(z) = \frac{1}{z} \int_0^z \frac{f(\zeta)}{1-\zeta}\, d\zeta,$$

which is of the form (1) for $A(\zeta) = 0$, $B(\zeta) = 1-\zeta$, $x_1 = x_2 = 0$, $\lambda_1 = 0$ and $\lambda_2 = 1$, or the Hilbert integral operator (as defined in [**2, 3**]),

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