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A CLASS OF INTEGRAL EQUATIONS AND INDEX TRANSFORMATIONS RELATED TO THE MODIFIED AND INCOMPLETE BESSEL FUNCTIONS

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ABSTRACT. We consider a parametric family of integral equations of the first kind, which can be treated as index transformations and generalize classical Kontorovich-Lebedev transformation and related operators. The kernel of these equations is associated with the modified and incomplete Bessel functions and their derivatives with respect to an index. For certain kernels general solutions are found by using Sneddon's operational proof of the inversion formula for the Kontorovich-Lebedev transformation.

1. Introduction. Let f, g be complex-valued measurable functions defined on $\mathbf{R}_+$ and $\mathbf{R}$, respectively. We will deal here with the following integral equation of the first kind

$$\int_0^\infty S_n(x,y) f(x) \frac{dx}{x} = g(y), \quad y \in \mathbf{R}, \tag{1.1}$$

and the kernel $S_n(x,y)$ is defined by the formula

$$S_n(x,y) = \frac{1}{2} \int_{-\infty}^{\infty} e^{-x \cosh u} P_n(u) e^{iuy}\, du, \quad n \in \mathbf{N}_0. \tag{1.2}$$

Here $P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$ is a polynomial of degree $n \in \mathbf{N}_0$ with real-valued coefficients. A function $g(y)$ in (1.1) is given

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