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THE GENERALIZED EULER-MACLAURIN FORMULA FOR THE NUMERICAL SOLUTION OF ABEL-TYPE INTEGRAL EQUATIONS

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ABSTRACT. An extension of the Euler-Maclaurin formula to singular integrals was introduced by Navot [**12**]. In this paper this result is applied to derive a quadrature rule for integral equations of the Abel type. We present a stability and convergence analysis and numerical results that are in good agreement with the theory. The method is particularly useful when combined with fast methods for evaluating integral operators.

1. Introduction. The generalized Abel equation is a weakly singular Volterra integral equation of the first kind which usually appears in the form

$$\int_0^t (t-\tau)^{-\alpha} k(t,\tau) g(\tau)\, d\tau = f(t). \tag{1}$$

Here $0 \le t \le T$, $0 < \alpha < 1$, and the kernel $k(\cdot,\cdot)$ is smooth and satisfies $k(t,t) = 1$. The following solvability result was given by Atkinson [**1**].

Theorem 1.1. *If $f(t) = t^{1-\alpha-\beta}\tilde{f}(t)$ with $\tilde{f} \in C^\infty[0,T]$ and $\beta < 1$, then (1) has a unique solution g which is of the form $g(t) = t^{-\beta}\widetilde{g}(t)$ and $\widetilde{g} \in C^\infty[0,T]$.*

Numerical methods for (1) are usually based on collocation or product integration methods. This subject has been studied extensively and is reviewed in the monographs by Linz [**6**] and Brunner [**2**]. A more recent approach to discretizing (1) is the use of the convolution quadrature rules [**9**].

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