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PROJECTION METHODS FOR FREDHOLM INTEGRAL EQUATIONS ON THE REAL SEMIAXIS

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ABSTRACT. Numerical procedures to solve Fredholm integral equations of the second kind on the real semiaxis are proposed. Their stability and convergence are proved and error estimates in L^p weighted norm are given. Numerical examples are also included.

1. Introduction. Let us consider Fredholm integral equations of the second kind on unbounded intervals of the following type

$$f(y) - \int_0^\infty k(x,y) f(x) w(x) dx = g(y), \tag{1.1}$$

where $w(x) = x^\alpha e^{-x^\beta}$, $\alpha > -1$, $\beta > 1/2$, $k(x,y)$ and $g(x)$ are known functions and $f(x)$ is an unknown function. We want to study equation (1.1) in the spaces L^p_u, $1 < p < \infty$ and $u(x) = x^\gamma e^{-x^\beta/2}$, $\gamma > -1/p$.

When $y \in (-1,1)$, w is a Jacobi weight and the integral is defined on the bounded interval $(-1,1)$, there is a large literature about the numerical solution of such kind of equations (see, for example [**1, 3, 6, 17**]). In [**5**] Fredholm integral equations of the second kind on the interval $(0,+\infty)$ are considered. Here the weight $w(x)$ is the classical Laguerre weight ($\beta = 1$) and the space is $L^2_{\sqrt{w}}$.

In this paper both $w(x)$ and $u(x)$ are more general weights. Moreover the index p can assume any real value in $(1,+\infty)$. The main difficulties

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