JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 21, Number 4, Winter 2009

SOLUTIONS OF A SYSTEM OF INTEGRAL EQUATIONS IN ORLICZ SPACES

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Communicated by Paul Eggermont

ABSTRACT. We consider the following system of integral equations

$$u_i(t) = \int_0^1 g_i(t,s) f_i(s, u_1(s), u_2(s), \dots, u_n(s))\, ds,$$
$$\text{a.e. } t \in [0,1], \quad 1 \le i \le n.$$

Our aim is to establish criteria such that the above system has a solution $(u_1, u_2, \dots, u_n)$ where $u_i \in L_\phi$ (Orlicz space), $1 \le i \le n$. We further investigate the system

$$u_i(t) = \int_0^1 g_i(t,s) H(s, u_1(s), u_2(s), \dots, u_n(s))\, ds,$$
$$\text{a.e. } t \in [0,1], \quad 1 \le i \le n$$

and establish the existence of *constant-sign* solutions in Orlicz spaces, i.e., for each $1 \le i \le n$, $\theta u_i \ge 0$ and $u_i \in L_\phi$, where $\theta \in \{1, -1\}$ is fixed.

1. Introduction. Let $x = (x_1, x_2, \dots, x_N)^T$ and $y = (y_1, y_2, \dots, y_N)^T$ be in $\mathbf{R}^N$. Throughout, by $x \ge y$ we shall mean $x_i \ge y_i$ for each $1 \le i \le N$. Similarly, if $x, y \in \mathbf{R}^{N \times N}$ (real $N \times N$ matrices), then $x \ge y$ also means inequality in the componentwise sense.

In this paper we shall consider the following systems of Hammerstein integral equations

$$u_i(t) = \int_0^1 g_i(t,s) f_i(s, u_1(s), u_2(s), \dots, u_n(s))\, ds, \quad \text{a.e. } t \in [0,1], \quad 1 \le i \le n \tag{1.1}$$

2000 AMS *Mathematics subject classification.* Primary 45G05, 45G15, 45M20.
Keywords and phrases. System of integral equations, Orlicz spaces, constant-sign solutions.
Received by the editors on June 21, 2006, and in revised form on November 7, 2007.
DOI:10.1216/JIE-2009-21-4-469