JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 21, Number 3, Fall 2009

A NOTE ON THE POLYNOMIAL APPROXIMATION OF VERTEX SINGULARITIES IN THE BOUNDARY ELEMENT METHOD IN THREE DIMENSIONS

ALEXEI BESPALOV

Communicated by Simon Chandler-Wilde, Ivan Graham and J. Trevelyan

JIEA: A special Issue for the UKBIM6 Meeting

ABSTRACT. We study polynomial approximations of vertex singularities of the type $r^\lambda|\log r|^\beta$ on three-dimensional surfaces. The analysis focuses on the case when $\lambda > -\frac{1}{2}$. This assumption is a minimum requirement to guarantee that the above singular function is in the energy space for boundary integral equations with hypersingular operators. Furthermore, such strong vertex singularities may appear in solutions to boundary integral formulations of time-harmonic problems of electromagnetism in Lipschitz domains. Thus, the approximation results for such singularities are needed for the error analysis of boundary element methods in three dimensions. Moreover, to our knowledge, the approximation of strong singularities ($-\frac{1}{2} < \lambda \leq 0$) by high-order polynomials is missing in the existing literature. In this note we prove an estimate for the error of polynomial approximation of the above vertex singularities on quasi-uniform meshes discretising a polyhedral surface. The estimate gives an upper bound for the error in terms of the mesh size h and the polynomial degree p.

1. Introduction. In this note we analyse polynomial approximations of vertex singularities inherent to solutions of boundary integral equations (BIE) on a Lipschitz polyhedral surface Γ. In particular, denoting by r the distance to a vertex of Γ, we study approximations

2000 AMS *Mathematics subject classification.* Primary 41A10, 65N15, 65N38.
Keywords and phrases. p-approximation, hp-approximation on quasi-uniform meshes, boundary element method, singularities
Supported by the INTAS Young Scientist Fellowship grant (project no. 06-1000014-5945) and by the Russian Science Support Foundation.
Supported by EPSRC under grant no. EP/E058094/1.
Received by the editors on December 11, 2007, and in revised form on July 14, 2008.
DOI:10.1216/JIE-2009-21-3-359