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A NEW RESULT ON THE SINGULAR VALUE ASYMPTOTICS OF INTEGRATION OPERATORS WITH WEIGHTS

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This paper is dedicated to Professor Rainer Kress on the occasion of his 65th birthday.

ABSTRACT. It is an interesting question for the analysis of linear ill-posed operator equations $Ax = y$ and it seems to be of some importance for regularization theory whether a non-compact linear operator with non-closed range applied to a compact linear operator mapping between Hilbert spaces can alter the degree of ill-posedness determined by the singular value decay rate $\sigma_n(A) \to 0$ as $n \to \infty$ of the compact operator A. For giving some more answer to that question we work in the space $L^2(0,1)$ and focus on non-compact multiplication operators M applied to the integration operator J such that $A = M \circ J$ determines the operator governing the equation. Compositions of this type occur as linearizations of different nonlinear inverse problems in natural sciences, engineering, and finance. Specifically, we are interested in the case of multiplication operators M generated by a multiplier function m having an essential zero in $[0,1]$. In particular, in a toy problem of inverse option pricing multipliers m with exponential-type zeros occur. By analyzing the strength of source conditions for obtaining convergence rates in regularization it was conjectured that the ill-posedness situation tends to the worse in the exponential case compared to the case of power-type zeros in m, for which we have shown in [**9**] that the degree of ill-posedness is uniformly one. Now we are going to extend this result to some family of exponential weight functions m and prove that the asymptotics $\sigma_n(A) \asymp n^{-1}$ also holds for

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