

THE COMPUTATION OF LOWER BOUNDS FOR THE NORM OF THE INDEX OF REFRACTION IN AN ANISOTROPIC MEDIA FROM FAR FIELD DATA

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*Dedicated to Professor Dr. Rainer Kress on the occasion of his
65th birthday and the pleasure that knowing him has given to our lives!*

ABSTRACT. We consider the scattering of time harmonic electromagnetic plane waves by a bounded, inhomogeneous, anisotropic dielectric medium and show that under certain assumptions a lower bound on the norm of the (matrix) index of refraction can be obtained from a knowledge of the smallest transmission eigenvalue corresponding to the medium. Numerical examples are given showing the efficaciousness of our estimates.

1. Introduction. Anisotropic material play a special role in electromagnetic inverse scattering theory. This is due to the fact that from far field data only the support D of the scatterer is uniquely determined [3], [15] and little can be said about the material properties of the scatterer [13]. This remains true even if multifrequency data is used. Although specific information about the material properties may be unavailable, there remains the possibility of obtaining upper or lower bounds on certain norms of the (matrix) index of refraction and it is to this task that this paper is directed. In particular, are there certain inequalities that the index of refraction must satisfy for a given measured far field pattern? For the case of a dielectric isotropic

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