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INTEGRO-DIFFERENTIAL EQUATIONS OF FIRST ORDER WITH AUTOCONVOLUTION INTEGRAL

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ABSTRACT. In the paper two classes of first order integro-differential equations with autoconvolution integral are studied generalizing an equation of J. M. Burgers from the turbulence theory. General existence and stability theorems in a finite interval are proved and the asymptotic behavior of the solutions at infinity is discussed.

1. Introduction. In his theory of turbulence J. M. Burgers [**3**] (for Burgers' turbulence see also [**6, 7, 12**]) studied an integro-differential equation which can be reduced to the equation

$$y'(x) + \left(\frac{1}{2x} - \frac{1}{16}x^2\right) y(x) = \int_0^x y(\xi)y(x-\xi)d\xi\,, \quad x > 0 \tag{1.1}$$

with autoconvolution integral $I(y) = \int_0^x y(\xi)y(x-\xi)d\xi$ and derived a solution of this equation by series expansions in powers and exponentials.

In this paper we deal with a general first order integro-differential equation of the form

$$y'(x) + k(x)y(x) = \int_0^x a(x,\xi)y(\xi)y(x-\xi)d\xi + \int_0^x b(x,\xi)y(\xi)d\xi + g(x), \quad x \in (0,T) \tag{1.2}$$

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