

**SOLVABILITY OF THREE POINT BOUNDARY VALUE  
PROBLEMS FOR SECOND ORDER INTEGRO-  
DIFFERENTIAL EQUATIONS OF MIXED TYPE**

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**ABSTRACT.** This paper investigates the existence of extreme solutions of the three point boundary value problem for a class of second order integro-differential equations of mixed type. By using the method of upper and lower solutions and monotone iterative technique, we establish the existence results of extreme solutions. An example is also provided to illustrate the efficiency of the obtained results.

**1. Introduction.** Theory of integro-differential equations in the field of modern applied mathematics has made considerable headway, because all the structure of its emergence has deep physical background and realistic mathematical model (see [1, 2]). One of the ideas in the study of certain higher order boundary value problems for differential equations is to reduce them to boundary value problems for lower order integro-differential equations [3, 4]. During the past years, many authors have paid attention to the research of three point boundary value problems for second order differential equations because of its potential applications, see, for example, [1, 5-7, 16]. In [8, 9], J.J. Nieto and R. Rodriguez-Lopez introduced a new concept of lower and upper solutions, they consider the periodic boundary value problems for the following first order functional differential equation

$$(1.1) \quad \begin{cases} u'(t) = g(t, u(t), u(\theta(t))), & t \in [0, T], \\ u(0) = u(T). \end{cases}$$

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