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A PARAMETER CHOICE STRATEGY FOR A MULTI-LEVEL AUGMENTATION METHOD SOLVING ILL-POSED OPERATOR EQUATIONS

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ABSTRACT. We apply the multi-level augmentation method for solving operator equations of the first kind via the Tikhonov regularization method. We present a new a posteriori parameter choice strategy which leads to optimal convergence rates. Numerical experiments illustrate the efficiency of the method.

1. Introduction. In this paper we consider the problem of solving the first kind operator equation

$$\mathcal{K}x = y, \tag{1.1}$$

where $\mathcal{K}$ is a linear compact operator from a Hilbert space $\mathbf{X}$ to another Hilbert space $\mathbf{Y}$. Assume that $y \in D(\mathcal{K}^\dagger) := R(\mathcal{K}) + R(\mathcal{K})^\perp$, where $R(\mathcal{K})$ is the range of the operator $\mathcal{K}$, and $\mathcal{K}^\dagger$ the Moore-Penrose generalized inverse of $\mathcal{K}$. It is known that the minimum norm least squares solution $x_* = \mathcal{K}^\dagger y$ of the equation (1.1) exists. In the following, for simplicity, we assume $y \in R(\mathcal{K})$, cf. [**6**].

In practice, only approximate righthand side $y^\delta \in \mathbf{Y}$ with $\|y - y^\delta\| \leq \delta$ is available, where $\delta > 0$ is an error level. Thus we need to solve the perturbed operator equation

$$\mathcal{K}x^\delta = y^\delta. \tag{1.2}$$

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