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# THE BEDROSIAN IDENTITY AND HOMOGENEOUS SEMI-CONVOLUTION EQUATIONS

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ABSTRACT. We characterize a class of functions satisfying the classical Bedrosian identity or the circular Bedrosian identity by certain homogeneous semi-convolution equations. The structure of solutions of these equations is then studied using translation invariant subspaces of Hardy spaces and additive positive definite kernels. The results obtained provide some insight into the Bedrosian identity and a construction of intrinsic mode functions for the time-frequency analysis of nonlinear and nonstationary signals.

**1. Introduction.** The Hilbert transform is defined for each function $f \in L^p(\mathbf{R})$, $1 \le p \le \infty$, at $x \in \mathbf{R}$ as

$$(1.1)\quad (Hf)(x) := \text{p.v.}\,\frac{1}{\pi}\int_{\mathbf{R}} \frac{f(y)}{x-y}\,dy := \lim_{\substack{\varepsilon\to 0^+\\ N\to\infty}} \frac{1}{\pi}\int_{\varepsilon\le|y-x|\le N} \frac{f(y)}{x-y}\,dy,$$

whenever the Cauchy principal value of the above singular integral exists. In engineering analysis, people often face the need for calculating the Hilbert transform of a product of functions. A simple method for computing such products under certain conditions was found by Bedrosian [**2**]: If two functions $f, g \in L^2(\mathbf{R})$ satisfy either $\operatorname{supp} \hat{f} \subseteq$

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