JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 20, Number 4, Winter 2008

A REGULARITY THEOREM FOR A VOLTERRA INTEGRAL EQUATION OF THE THIRD KIND

PETER GRANDITS

Communicated by S. Londen

ABSTRACT. An existence and smoothness theorem is given for a Volterra integral equation of the form

$$f(x)v(x) = \phi(x) - \int_0^x K(x,\xi)v(\xi)\,d\xi,$$

where $f(x)$ has a zero at $x = 0$, and the kernel $K(x,\xi)$ has a kind of square root behavior at the diagonal $x = \xi$.

1. Introduction. In this paper we will consider a special class of third kind linear Volterra integral equation, i.e.

$$f(x)v(x) = \phi(x) - \int_0^x K(x,\xi)v(\xi)\,d\xi. \tag{1}$$

It is easy to see that this type of equation can be written as an equation of the second kind ($f(x) \equiv 1$), which, in general, has a singular kernel, if the function $f(x)$ has zeroes. Seminal works dealing with this type of equations are [5] and [6]. The idea of [6] is to split up the kernel $K(x,\xi)$ into a constant part, w.l.o.g. one can take 1, and a function $\Gamma(x,\xi)$. Evans constructed then an approximating series for the solution. Each term of this series is constructed in two steps (for more details see the proof of Proposition 2.1 below) : First one has to solve a singular differential equation, stemming from the constant part of the kernel; in the second step one has to evaluate an integral in order to get the

Keywords and phrases. third kind Volterra integral equation, regularity
Received by the editors on August 8, 2006, and in revised form on January 5, 2007.

DOI:10.1216/JIE-2008-20-4-507