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CLOSED-FORM ERROR ESTIMATES FOR THE NUMERICAL SOLUTION OF FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. Closed-form algebraic formulae are derived for the local error incurred in the numerical solution of integral equations by iterated collocation methods, the analysis being illustrated by application to Fredholm integral equations of the second kind. The novel error analysis uses an asymptotic approach, in the small parameter of the numerical mesh size, applied to a finite-rank degenerate-kernel orthogonal-polynomial approximation of the exact kernel. It is proved that, under suitable conditions, the discrepancy between our theoretically predicted error and the actual numerical error tends to zero at the rate $\|\mathcal{K} - \mathcal{K}_M\|$, where M is the rank of the degenerate-kernel approximation. A leading-order error analysis is validated on three increasingly accurate projection methods applied to both smooth and sharply peaked kernels, our error predictions being demonstrated to be exponentially convergent to experimentally obtained global numerical errors. The new method is demonstrated to be cost-effective relative to standard extrapolation.

1. Introduction. This paper is concerned with obtaining explicit, closed-form, algebraic expressions for the error incurred in the numerical solution of the Fredholm integral equation of the second kind,

$$\phi(x) = f(x) + \lambda \int_a^b K(x,s)\,\phi(s)\,ds\ , \tag{1}$$

for a variety of approximating projection methods. Although degenerate-kernel (and orthogonal-polynomial) techniques do indeed constitute an

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