

WEIGHTED GENERALIZED HÖLDER SPACES
AS WELL-POSEDNESS CLASSES
FOR SONINE INTEGRAL EQUATIONS

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ABSTRACT. For integral equations of the first kind

$$\mathbb{K}\varphi := \int_0^x k(x-t)\varphi(t) dt = f(x), \quad x \in (0, b)$$

where $0 < b < \infty$, in the case of a certain class of almost decreasing Sonine kernels $k(t)$ we prove weighted estimates of continuity moduli $\omega(\mathbb{K}\varphi, h)$ and $\omega(\mathbb{K}^{-1}f, h)$. This allows us to show that the weighted generalized Hölder spaces $H^\omega(\rho)$ and $H^{\omega_1}(\rho)$ are suitable well-posedness classes for these integral equations of the first kind under the choice $\omega_1(h) = hk(h)\omega(h)$.

1. Introduction. We consider integral equations of the first kind

$$(1.1) \quad \mathbb{K}\varphi := \int_0^x k(x-t)\varphi(t) dt = f(x), \quad x \in (0, b),$$

where $0 < b < \infty$, and $k(x) \in L_1(0, b)$.

As is well known, one of the main problems for integral equations of the first kind is to find "nice" well-posedness classes. Spaces of integrable functions do not suit well for this purpose in the following

Key words and phrases: integral equations of the first kind, Sonine kernels, modulus of continuity, Zygmund type estimate, weighted generalized Hölder spaces, almost decreasing and almost increasing functions

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