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PONTRYAGIN PRINCIPLE IN ABSTRACT SPACES

B. D. CRAVEN

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ABSTRACT. Pontryagin's theory for an optimal control problem with dynamics described by an ODE has possible extensions to other systems, such as some PDEs. If the system with state x and control u is described abstractly by $Dx = M(x, u)$, then some other linear mappings D than differentiation can lead to a Pontryagin principle. Costate boundary conditions are obtained by calculating an adjoint mapping. If the domain is not compact, but the problem reaches a strict minimum, then under some continuity restrictions the control problem can be approximated closely by one for which Pontryagin's principle holds.

1. Introduction. The optimal control problem:

$$\text{MIN}_{x,u}\ F(x,u) := \int_0^T f(x(t),u(t),t)dt \text{ subject to}$$
$$x(0) = x_0,\ \dot{x}(t) = m(x(t),u(t),t)\ (0 \le t \le T),$$
$$u(t) \in \Gamma(t)\ (0 \le t \le T)\ \Leftrightarrow (\forall t) g(x(t),t) \le 0$$

may be written as:

$$\text{MIN}_{x,u}\ F(x,u) \text{ subject to } Dx = M(x,u),$$

where $Dx = w \Leftrightarrow x = x_0 + \int_0^t w(s)ds,\ M(x,u)(t) := m(x(t),u(t),t)$, and D is made continuous by giving a suitable graph norm to the space X of states. This formulation suggests a generalization in which the domain $[0,T]$ is replaced by a closed subset $E \subset \mathbf{R}^r$ $(r \ge 1)$,

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