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CONSTANT-SIGN SOLUTIONS OF A SYSTEM OF VOLTERRA INTEGRAL EQUATIONS IN ORLICZ SPACES

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This paper is dedicated to Professor Zuhair Nashed in recognition of his contributions to the field of integral and operator equations.

ABSTRACT. We consider the following system of Volterra intergral equations

$$u_i(t) = \int_0^t g_i(t,s) f_i(s, u_1(s), u_2(s), \cdots, u_n(s)) ds,$$
$$a.e.\ t \in [0,T],\ 1 \le i \le n.$$

Criteria are offered for the existence of one and more *constant-sign* solutions $u = (u_1, u_2, \cdots, u_n)$ of the system in L^p and the Orlicz spaces. We say u is of *constant sign* if for each $1 \le i \le n$, $\theta_i u_i(t) \ge 0$ for *a.e.* $t \in [0,T]$, where $\theta_i \in \{1, -1\}$ is fixed.

1. Introduction. In this paper we shall consider the system of Volterra integral equations

$$u_i(t) = \int_0^t g_i(t,s) f_i(s, u_1(s), u_2(s), \cdots, u_n(s)) ds, \quad a.e.\ t \in [0,T],\ 1 \le i \le n. \tag{1.1}$$

Throughout, let $u = (u_1, u_2, \cdots, u_n)$. We are interested in establishing the existence of one and more solutions u of the system (1.1) in

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