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OBLIQUE DUALS ASSOCIATED WITH RATIONAL SUBSPACE GABOR FRAMES

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ABSTRACT. Given parameters $a, b > 0$ such that the product $a\,b$ is rational, we consider subspace Gabor frames $\mathbf{G}(b, a, g)$ and $\mathbf{G}(b, a, k)$ and ask for conditions under which the subspace generated by the second frame contains a function which acts as a Gabor dual for the first one ("oblique dual"). A necessary and sufficient condition for the existence of such a function is given in terms of an inequality between certain matrix-valued functions constructed using the Zak transform of the generators g and k. The uniqueness of the oblique dual is also characterized.

1. Introduction. The theory of frames, generalizing the notion an orthonormal basis in a Hilbert space to systems that might be overcomplete, was first introduced by R. J. Duffin and A. C. Schaeffer in [8] (see also [18]). We briefly recall some terminology, notation and basic facts about this tool which plays an important role in modern theories such as that of wavelet and Gabor expansions. If $\mathcal{N}$ is a countable index set and $\mathcal{H}$ is an infinite-dimensional separable Hilbert space with inner product $\langle ., . \rangle$, we say that a collection $X = \{x_n\}_{n\in\mathcal{N}}$ in $\mathcal{H}$ is a *subspace frame* (for its closed linear span $\mathcal{M}$) if there exist constants $C_1, C_2 > 0$, called the *frame bounds*, such that

$$C_1 \,\|x\|^2 \leq \sum_{n\in\mathcal{N}} |\langle x, x_n\rangle|^2 \leq C_2 \,\|x\|^2, \quad x \in \mathcal{M}. \tag{1.1}$$

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