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CONVERGENCE RATES OF A MULTILEVEL METHOD FOR THE REGULARIZATION OF NONLINEAR ILL-POSED PROBLEMS

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ABSTRACT. In this paper, we prove convergence rates for a previously [22] proposed multilevel method for solving nonlinear ill-posed operator equations

$$F(x) = y.$$

By minimizing the distance to some initial guess under the constraint of a discretized version of the operator equation for different levels of discretization, we define a sequence of regularized approximations to the exact solution, that in [22] had been shown to be stable and convergent for arbitrary initial guess, and can be computed via a multilevel procedure that altogether yields a globally convergent method. In the present paper we prove optimal logarithmic and Hölder type convergence rates under respective source conditions. Moreover we provide a tool for possible numerical solution strategies for the minimization problem on each level of discretization by providing an exact penalty function derived via an augmented Lagrangian approach.

1. Introduction. Consider a nonlinear operator equation

$$F(x) = y \tag{1}$$

with a continuous operator $F : \mathcal{D}(F) \subseteq X \to Y$ between Hilbert spaces X, Y, that is ill-posed in the sense of unstable dependence of a solution

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