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FAST COLLOCATION METHODS FOR HIGH-DIMENSIONAL WEAKLY SINGULAR INTEGRAL EQUATIONS

ZHONGYING CHEN, BIN WU AND YUESHENG XU

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Dedicated to Professor Qun Lin on the occasion of his 70th birthday with friendship and esteem

ABSTRACT. We realize fast collocation methods for solving Fredholm integral equations of the second kind with weakly singular kernels on a polyhedral domain in $\mathbf{R}^d$ with $d \geq 3$. A polyhedral domain is subdivided into a finite number of simplices. We construct a uniform self-similar partition of a simplex for the purpose of constructing multi-scale bases and their corresponding collocation functionals. The multi-scale bases and the collocation functionals lead to a compression of the matrix representation of the weakly singular integral operator and thus to a fast collocation scheme for solving the integral equation. We develop a quadrature rule for computing the weakly singular integrals appearing in the matrix.

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