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BOUNDARY INTEGRAL EQUATIONS ON UNBOUNDED ROUGH SURFACES: FREDHOLMNESS AND THE FINITE SECTION METHOD

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ABSTRACT. We consider a class of boundary integral equations that arise in the study of strongly elliptic BVPs in unbounded domains of the form $D = \{(x, z) \in \mathbf{R}^n \times \mathbf{R} : z > f(x)\}$ where $f : \mathbf{R}^n \to \mathbf{R}$ is a sufficiently smooth bounded and continuous function. A number of specific problems of this type, for example, acoustic scattering problems, problems involving elastic waves and problems in potential theory, have been reformulated as second kind integral equations $u + Ku = v$ in the space BC of bounded, continuous functions. Having recourse to the so-called limit operator method, we address two questions for the operator $A = I + K$ under consideration, with an emphasis on the function space setting BC. Firstly, under which conditions is A a Fredholm operator, and, secondly, when is the finite section method applicable to A?

1. Introduction. The boundary integral equation method is very well developed as a tool for the analysis and numerical solution of strongly elliptic boundary value problems in both bounded and unbounded domains, provided the boundary itself is bounded, e.g., [**6, 29, 38**].

In the case when both domain and boundary are unbounded, the theory of the boundary integral equation method is much less well developed. The reason for this is fairly clear, namely, that loss of compactness of the boundary leads to loss of compactness of boundary integral operators. To be more precise, classical applications of the

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