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SOLUTION OF THE GENERALIZED ABEL INTEGRAL EQUATION

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ABSTRACT. A direct function theoretic method is employed to determine the closed form solution of the generalized Abel integral equation. The present form of the solution involves only weakly singular integrals to be evaluated finally as opposed to the known form that requires evaluation of strongly singular integrals of the Cauchy type.

1. Introduction. The generalized Abel integral equation

$$(1) \quad a(x)\int_{\alpha}^{x}\frac{\phi(t)\,dt}{(x-t)^{u}}+b(x)\int_{x}^{\beta}\frac{\phi(t)\,dt}{(t-x)^{u}}=f(x), \qquad (0<\mu<1) \quad (\alpha\le x\le\beta)$$

where the coefficients $a(x)$ and $b(x)$ do not vanish simultaneously, is solved in closed form, under the specific assumptions on the functions a, b, f and ϕ, though not stated explicitly, which will be clear from the form of the solutions derived later on.

The generalized Abel equation (1) was examined in Gakhovs book [1], under the special assumptions that the coefficients $a(x)$ and $b(x)$ satisfy Hölders conditon in $[\alpha,\beta]$, whereas the forcing term $f(x)$ and the unknown function $\phi(x)$ belong to those classes of functions which admit representations of the form

$$(2) \quad \left.\begin{aligned} f(x) &= [(x-\alpha)(\beta-x)]^{\epsilon}\, f^{*}(x), \\ \text{and}\quad \phi(x) &= \frac{\phi^{*}(x)}{[(x-\alpha)(\beta-x)]^{1-\mu-\epsilon}} \end{aligned}\right\} \quad (\epsilon>0)$$

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