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STABILIZABILITY OF INTEGRODIFFERENTIAL PARABOLIC EQUATIONS

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ABSTRACT. We consider the stabilizability problem for an abstract parabolic integrodifferential equation. Under suitable assumptions, we give a necessary and sufficient condition for stabilizability, generalizing the well known Hautus condition. Then we apply the abstract result to parabolic integrodifferential equations in bounded domains.

Introduction. We consider a parabolic integrodifferential equation in general Banach space X:

$$(0.1)\quad u'(t) = Au(t) + \int_0^t K(t-s)u(s)ds + \Phi f(s), \quad t > 0, u(0) = u_0.$$

Here $A : D(A) \to X$ generates an analytic semigroup, and $K : [0, +\infty[\to \mathrm{L}(D(A), X)$ is a Laplace transformable function. $\Phi \in L(Y, X)$, where Y is a Banach space. Other assumptions are made in order that a spectrum determining condition holds and that the theory developed in [**2**] is applicable.

Roughly speaking, the "resolvent set" in integrodifferential equations of this kind is the set of all $\lambda_0 \in \mathbf{C}$ such that the function $\lambda \to (\lambda - A - \hat{K}(\lambda))^{-1}$ either is well defined or has an analytic extension at λ_0 ($\hat{K}$ is the Laplace transform of the function K). Its complementary set σ is the "spectrum" for problem (0.1).

If $\sup\{\mathrm{Re}\,\lambda : \lambda \in \sigma\} < 0$, then the free system (with $f \equiv 0$) is exponentially stable: all the solutions decay exponentially to 0 as $t \to +\infty$. If $\sup\{\mathrm{Re}\,\lambda : \lambda \in \sigma\} \geq 0$, we consider the following problem: find conditions on Φ ensuring that, for each initial value u_0, there exists f such that the solution u of (0.1) converges asymptotically to zero (preferably exponentially, and in the graph norm of A) as $t \to +\infty$. If this happens, system (0.1) is said to be stabilizable. We are also interested in the exponential decay of Cu, where the "observation

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