JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 2, Spring 1990

ON AN EXTENSION OF THE TROTTER-KATO THEOREM FOR RESOLVENT FAMILIES OF OPERATORS

C. LIZAMA

ABSTRACT. We extend the well-known theorem on convergence and approximation of C_0-semigroups due to Trotter and Kato to the context of resolvent families of operators. In particular, this result also extends those due to Goldstein [**4**] for cosine families and it is applied to the case of a class of Volterra equations which were considered by Prüss [**11**].

1. Introduction. Let X be a Banach space with norm $\|\ \|$. Let A be an unbounded closed linear operator in X, with dense domain $D(A)$, and $k \in L^1_{\text{loc}}(\mathbf{R}_+)$.

A strongly continuous family $\{R(t), t \geq 0\}$ of bounded linear operators in X is called a *resolvent family* (for equation (1.2) below) if it commutes with A and satisfies the resolvent equation

$$(1.1) \qquad R(t)x = x + \int_0^t k(t-s)AR(s)x\,ds, \quad t \geq 0, x \in D(A).$$

We remark that a resolvent family is unique if it exists (cf. [**3**]).

The notion of resolvent family is the natural extension of the concepts of a C_0-semigroup for $k(t) \equiv 1$ and of a cosine family for the case $k(t) \equiv t$.

The existence of a resolvent family allows one to solve the Volterra equation

$$(1.2) \quad u(t) = f(t) + \int_0^t k(t-s)Au(s)ds, \quad t \in [0,T] =: J,\ f \in C(J,X).$$

Equation (1.2) has been considered recently by many authors, since it has applications in different fields (see, for example, [**3, 11, 12**]).

The following generation theorem, due to Da Prato and Iannelli [**2**], is the extension for resolvent families of the well-known generation