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SPECTRAL APPROXIMATIONS FOR WIENER-HOPF OPERATORS

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ABSTRACT. The main purpose of this paper is to compare spectral properties of a Wiener-Hopf operator

$$Kf(s) = \int_0^\infty \kappa(s-t)f(t)\,dt$$

and corresponding finite-section operators

$$K_\beta f(s) = \int_0^\beta \kappa(s-t)f(t)\,dt,$$

where $\kappa \in L^1(R)$ and f is bounded and continuous. Among other results, we show that any neighborhood of the spectrum of K contains the spectrum of K_β for β sufficiently large. However, the roles of K and K_β cannot be reversed. Examples are given with $\sigma(K)$ a disc and $\sigma(K_\beta) = \{0\}$ for all β. We also compare spectral properties of K_β and corresponding numerical-integral operators $K_{\beta n}$. The spectral properties of K_β and $K_{\beta n}$ match more completely than do the spectral properties of K and K_β.

1. Introduction. Let K be a Wiener-Hopf operator,

$$Kf(s) = \int_0^\infty \kappa(s-t)f(t)\,dt, \quad s \in \mathbf{R}^+,$$

where $\kappa \in L^1(\mathbf{R})$ and $f \in X^+$, the space of bounded, continuous, complex-valued functions on $\mathbf{R}^+$ with the uniform norm $||f|| = \sup|f(t)|$. Corresponding finite-section operators K_β are given by

$$K_\beta f(s) = \int_0^\beta \kappa(s-t)f(t)\,dt, \quad s \in \mathbf{R}^+, \beta \in \mathbf{R}^+.$$

We shall compare spectral properties of K and K_β as $\beta \to \infty$. This continues a study of integral equations on the half line initiated in **[3]**