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ON THE STABLE AND UNSTABLE SUBSPACES OF A CRITICAL FUNCTIONAL DIFFERENTIAL EQUATION

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ABSTRACT. We study the asymptotic behavior of the linear, infinite delay, autonomous system of functional differential equations

$$(*)\qquad \begin{aligned} x'(t) + \mu * x(t) &= 0, & t \geq 0,\\ x(t) &= \phi(t), & t \leq 0. \end{aligned}$$

Here μ is an n-dimensional matrix-valued measure supported on $[0,\infty)$, finite with respect to a weight function, ϕ is a $\mathbf{C}^n$-valued continuous function in a fading memory space, and x is a locally absolutely continuous function for $t \geq 0$, satisfying $(*)$. We find conditions that ensure that the state space of $(*)$ can be written as a direct sum of a stable subspace, characterized by the fact that solutions are small at infinity, a finite dimensional central subspace in which solutions are neither small nor large at infinity, and a finite dimensional exponentially unstable subspace consisting of exponentially growing solutions. This work is heavily based on earlier joint work [**2**] with Jordan and Wheeler, and it extends the main result in [**3**]. The basic difference is that here we do not allow an explicit forcing term on the right-hand side of the first of the two equations in $(*)$, but instead we are able to relax the assumptions on the kernel.

1. Introduction. We study the asymptotic behavior of the solutions of the linear, infinite delay, autonomous system of functional differential equations

$$(1.1)\qquad \begin{aligned} x'(t) + \mu * x(t) &= 0, & t \in \mathbf{R}^+,\\ x(t) &= \phi(t), & t \in \mathbf{R}^-. \end{aligned}$$

Here $\mathbf{R}^+ = [0,\infty)$, $\mathbf{R}^- = (-\infty, 0]$, μ is an n by n matrix-valued measure supported on $\mathbf{R}^+$ which is finite with respect to a weight function, and

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