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INTEGRAL EQUATIONS FOR TRANSMISSION PROBLEMS IN LINEAR ELASTICITY

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ABSTRACT. The scattering of elastic waves by a penetrable homogeneous object is described by a system of integral equations for the field and its traction on the boundary of the scatterer. The system contains the operators of the single and double layer potentials, of the traction of the single layer potential, and of the traction of the double layer potential. It is a strongly elliptic system of pseudodifferential equations. Therefore every Galerkin scheme for its approximate solution is convergent. For Lipschitz boundaries we show strong ellipticity of the system of boundary integral operators. This implies existence and uniqueness of the solution and quasioptimal error estimates for its Galerkin solutions.

1. Introduction. A classical tool for the analysis of transmission problems is the reduction to boundary integral equations (see [**11**]). The property of *strong ellipticity* of boundary integral equations is known to be useful in several respects: It yields existence proofs as well as convergence results for approximate solutions. In the case of *boundary value problems*, this strong ellipticity has been analyzed extensively (see [**3**, **7**, **12**, **17**] and the literature quoted there). It turned out that there exist methods for constructing strongly elliptic boundary integral equations for general strongly elliptic boundary value problems and even for non-smooth (Lipschitz) boundaries in the case of second order systems. For *transmission problems*, in [**4**] the case of the Helmholtz equation was analyzed and a proof for strong ellipticity of the boundary integral equations on smooth boundaries and on plane polygonal boundaries was given. In the paper [**5**], the scattering of electromagnetic waves was studied and a general principle was found that allows proof of strong ellipticity for the system of integral equations which is obtained by application of the "direct method" to transmission problems. This principle yields strongly elliptic boundary integral equations for general combinations of boundary and transmission conditions [**13**].

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