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DISTRIBUTIONAL SOLUTIONS OF SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. It is proved that certain singular equations, which have no classical solutions because of singularities of the coefficients on the interval of integration, still have distributional solutions. The explicit form of these distributional solutions is presented.

1. Introduction. In an accompanying paper [**5**], Cauchy type singular integral equations over an interval $I \subseteq \mathbf{R}$ are solved by an orthogonality technique. The two equations considered are the direct equation (I-1) and the associated equation (I-1*). As was pointed out in [**5**], certain problems may arise in the solutions of these two equations when either of the functions defined by

$$\Lambda^{\pm}(t) = \lambda(t) \pm \pi i \eta(t)$$

has a zero. In certain cases these problems may be circumvented. In particular, if λ and η are real functions, then Λ^+ and Λ^- vanish at the same point(s), and the solution of (I-1) presents no problem. The same is true of (I-1*) if $g(t)$ has zero(s) at the same point(s) as $\Lambda^{\pm}$ (of order greater than or equal to the order of the zero(s) of $\Lambda^{\pm}$).

In case this condition on $g(t)$ is not satisfied, it was shown in [**5**] that a weak solution of (I-1*) could still be obtained under the condition that $\Lambda^{\pm}$ are the boundary values of an analytic function $\Lambda(z)$. This condition, while restrictive, is often satisfied in transport theory [1, 2], so this type of solution is not without practical application.

In the present note, we see (in §2) how in certain cases the distributional solutions to (I-1*) can still be obtained when the condition on the vanishing of $f(t)$ is not satisfied, even though no such function $\Lambda(z)$, as described above, exists. In §3 we present an application of the method developed in §2.

2. Distributional solutions of (I-1*). The method followed here was suggested by the analysis of a transport equation in which $\Lambda^{\pm}(t)$