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APPLICATION OF ORTHOGONALITY RELATIONS TO SINGULAR INTEGRAL EQUATIONS

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ABSTRACT. Singular integral equations with Cauchy type kernels are considered on a real interval. A generalization of the partial range orthogonality relations of neutron transport theory is used to generate solutions without the need of introducing the Hilbert transform, as in the standard treatment. This method has the advantage of clarifying the origin of the "endpoint conditions" in the solution of the Riemann-Hilbert problem, and, in addition, it simplifies the treatment of embedded eigenvalues.

1. Introduction. Singular integral equations of Cauchy type arise in a number of fields of physics and engineering; the general theory has been studied extensively and is described, for example, in books by Muskhelishvili [**13**], Gohberg and Krupnik [**9**] and Prößdorf [**15**] and a recent review paper by Estrada and Kanwal [8]. (A description of applications in transport theory has been given also, by Case [**4**].)

In this paper, we consider the case that the range of integration is the bounded closed interval $[-1,1]$. The simplest such equation can be written

$$f(t) = A(t)\lambda(t) + \mathcal{P}\int_{-1}^{1} \frac{A(\nu)\eta(\nu)}{\nu - t}\,d\nu, \qquad t \in (-1,1). \tag{1}$$

We assume that f, λ and η are real-valued uniformly Hölder continuous functions, although the extension to $f \in L_p[-1,1]$, $1 < p < \infty$, is straightforward. The methods we are going to describe can be adapted easily to more general Cauchy type equations, such as those described in Chapter 6 of [**13**] or in §4 of [**8**], but the general techniques are more easily understood in the simplest case which, anyway, covers many applications of physical interest.

In addition to (1), we shall discuss the associated equation

$$g(t) = B(t)\lambda(t) + \eta(t)\mathcal{P}\int_{-1}^{1} \frac{B(\nu)}{\nu - t}\,d\nu, \qquad t \in (-1,1). \tag{1*}$$