CONVERGENCE OF INNER/OUTER SOURCE ITERATIONS WITH FINITE TERMINATION OF THE INNER ITERATIONS

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ABSTRACT. A two-stage (nested) iteration strategy, in which the outer iteration is analogous to a block Gauss-Seidel method and the inner iteration to a Jacobi method for each of these blocks, often are used in the numerical solution of discretized approximations to the neutron transport equation. This paper is concerned with the effect, within a continuous space model, of errors from finite termination of the inner iterations upon the overall convergence. The main result is that convergence occurs, to the solution of the original problem, in the limit that both the outer iteration index and the minimum over all outer indices and all groups ("blocks") jointly become large. Positivity properties (in the sense of cone preserving) are used extensively.

1. Introduction. A first step in computationally solving energy dependent linear (steady state) particle transport problems often is to introduce so-called *outer* (energy/group) source iteration. Conceptually, each step within this iteration involves solution of a sequence of monoenergetic transport problems. It is widespread practice to solve these by *inner* (direction) source iteration. This paper is concerned with the effect of the (computationally necessary) finite termination of the inner iterations upon the convergence of the approximations produced by the outer iterations. It is self-contained regarding the technical aspects of these iterative processes, but the interested reader is referred to the monographs by Bell and Glasstone [1], Duderstadt and Martin [2], Lewis and Miller [3], and Marchuk and Lebedev [4] for further background in transport theory.

Suppose that the underlying "exact" transport problem is subcritical (satisfies assumption A.4 of the next section), and denote the corresponding angular flux (the vector of dependent variables) by $\psi = \psi(x)$. If $\psi^{(i)} = \psi^{(i)}(x)$ denotes the ith approximation from the outer source iteration, then it is relatively easy (see §II for an example) to show that

$$(1) \qquad \psi = \lim_{i \to \infty} \psi^{(i)},$$