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PROJECTION METHODS FOR SINGULAR INTEGRAL EQUATIONS

DAVID ELLIOTT

ABSTRACT. Both necessary and sufficient conditions are given under which the direct and indirect methods of finding the approximate solution of singular integral equations, with Cauchy kernel, are the same. The theory is applied to two examples and the paper concludes by considering the Sloan iteration applied to the direct method.

1. Introduction. We consider projection methods for the approximate solution of the singular integral equation

$$a(t)\phi(t) + \frac{b(t)}{\pi}\int_{-1}^{1}\frac{\phi(\tau)\,d\tau}{\tau - t} + \int_{-1}^{1}k(t,\tau)\phi(\tau)\,d\tau = f(t), \tag{1.1}$$

on the arc (-1,1). The first integral is to be interpreted as the Cauchy principal value. The functions a, b, k and f are given and the unknown function ϕ is required or, through the projection methods, approximations to ϕ. Rewrite (1.1) as

$$M\phi + K\phi = f \tag{1.2}$$

where

$$M\phi(t) = a(t)\phi(t) + \frac{b(t)}{\pi}\int_{-1}^{1}\frac{\phi(\tau)\,d\tau}{\tau - t} \tag{1.3}$$

and

$$K\phi(t) = \int_{-1}^{1}k(t,\tau)\phi(\tau)\,d\tau. \tag{1.4}$$

Suppose that the linear operators M and K each map a normed space X into a normed space Y. The spaces are chosen so that M is bounded and K compact. The function f is an element out of Y and $\phi \in X$.

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