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GLOBAL STABILITY CONDITION FOR COLLOCATION METHODS FOR VOLTERRA INTEGRAL EQUATIONS OF THE SECOND KIND

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ABSTRACT. The solution of the Volterra integral equation with degenerate kernel

$$y(t) = g(t) + \int_0^t \sum_{i=1}^n a_i(t) b_i(s) y(s)\, ds, \qquad t \geq 0,$$

is bounded provided that g and $\sum_{i=1}^n |a_i(t)|$ are bounded, and $b_j, j = 1, 2, \ldots, n$ are absolutely integrable.

It is shown that under the same hypotheses this property is inherited by the numerical solution resulting from applying exact collocation methods to this equation.

1. Introduction. The purpose of this paper is to investigate stability properties of exact collocation methods for Volterra integral equations (VIEs) of the second kind

$$y(t) = g(t) + \int_0^t k(t, s, y(s))\, ds, \qquad t \in [0, T], \tag{1}$$

where the functions g and k are continuous. We denote by Y the unique solution of this equation.

Consider the partition $0 = t_0 < t_1 < \cdots < t_N = T$ of the interval $[0, T]$, and put $h_i = t_{i+1} - t_i$, $\sigma_0 = [t_0, t_1], \sigma_i = (t_i, t_{i+1}], i = 1, 2, \ldots, N-1$, $Z_N = \{t_i : i = 1, 2, \ldots, N\}$. Define also the set X of collocation points by

$$X = \cup_{i=0}^{N-1} X_i,$$

where $X_i = \{t_{i,j} := t_i + c_j h_i,\ 0 \leq c_1 < c_2 < \cdots < c_m \leq 1\}$. Here, $c_j, j = 1, 2, \ldots, m$, are given collocation parameters.

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