JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 1, Winter 1989

# AN ITERATION PROCEDURE FOR A CLASS OF INTEGRODIFFERENTIAL EQUATIONS OF PARABOLIC TYPE

JOHN M. CHADAM AND HONG-MING YIN

ABSTRACT. This paper deals with a class of integrodifferential equations of parabolic type in which a function of the solution and its derivatives up to the second order with respect to the space variables is involved in a definite integral over the region. The problem can be applied to various models in physics and engineering. An iteration approach is used to establish the global solvability and stability for the problem. The technique is based on estimates of Green's function along with Gronwall's inequality.

**1. Introduction.** Let $T > 0$ and $Q_T = \Omega \times (0,T)$, where $\Omega$ is a bounded region in $R^n$ with a smooth boundary $\partial\Omega$. Consider the following initial-boundary value problem:

$$(1.1)\qquad Lu = h(x,t) + \int_\Omega B(x,t,u,u_x,u_{xx})\,dx, \qquad \textit{in } Q_T,$$

$$(1.2)\qquad u(x,t) = 0, \qquad (x,t) \in S_T = \partial\Omega \times (0,T),$$

$$(1.3)\qquad u(x,0) = u_0(x), \qquad x \in \bar{\Omega},$$

where

$$L = \frac{\partial}{\partial t} - \left[a_{ij}(x,t)\frac{\partial^2}{\partial x_i x_j} + b_i(x,t)\frac{\partial}{\partial x_i} + c(x,t)\right]$$

is a parabolic operator with $a_{ij}\xi_i\xi_j \geq a_0|\xi|^2 (a_0 > 0)$ for $\xi \in R^n$, while $u_x = \{u_{x_i}; i = 1,2,\cdots,n\}$ and $u_{xx} = \{u_{x_ix_j}; i,j = 1,2,\cdots,n\}$.

Recently, much attention has been given to the study of integrodifferential equation of the following evolution type

$$Lu = \int_0^t A(x,t,u,u_x,u_{xx})\,dt,$$

where $L$ is a parabolic operator. It represents a class of mathematical models which take into account the effect of the past history. Various

This work is partially funded by NSERC Canada.

Copyright ©1989 Rocky Mountain Mathematics Consortium