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POINTWISE ERROR ESTIMATES FOR THE TRIGONOMETRIC COLLOCATION METHOD APPLIED TO SINGULAR INTEGRAL EQUATIONS AND PERIODIC PSEUDODIFFERENTIAL EQUATIONS

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Dedicated to the memory of Professor Dr. P. Henrici.

ABSTRACT. Pointwise rates of convergence for the collocation method applied to periodic singular integral equations and pseudodifferential equations are considered, using trigonometric polynomials of degree n as the space of trial functions. If the exact solution is in C^r, then the error in the maximum norm is shown to be $O(n^{-r}\log^2 n)$. This rate of convergence is almost optimal, since the error for the interpolant of the exact solution is $O(n^{-r}\log n)$ and for the best approximation is $O(n^{-r})$.

Introduction. This paper deals with the trigonometric collocation method and is a sequel to [**7**], which treated the trigonometric Galerkin method. We prove error estimates of the form

$$\|u_n - u\|_{C^s} \le c(1/n)^{r-s}(\log n)^2\|u\|_{C^r}$$

for non-negative integers $s < r$, where u_n is the trigonometric polynomial of degree n obtained via collocation of a periodic singular integral equation or, more generally, of a periodic pseudodifferential equation, whose exact solution is u. For any periodic function $u \in C^r$, the error for the best approximation to u in the C^s norm by trigonometric polynomials of degree n is of order $(1/n)^{r-s}$. In this sense the error estimate above is less than optimal by a factor of $(\log n)^2$. Very recently, B. Silbermann (oral communication) has improved upon our result by showing that only one factor of $\log n$ is needed. This means that the

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