JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 2, Number 1, Winter 1989

COLLOCATION METHODS FOR SECOND KIND INTEGRAL EQUATIONS WITH NON-COMPACT OPERATORS

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ABSTRACT. We study the uniform convergence of collocation methods for integral equations on the half line, where the integral operator is a compact perturbation of a Wiener-Hopf operator. We prove that the collocation and the iterated collocation solutions converge to the exact solution with optimal orders of convergence, provided the meshes are appropriately graded to take account of the asymptotic behavior of the solution. As a consequence of the analysis similar optimal convergence results are proved for the case of boundary integral equations on polygonal domains.

1. Introduction. Initially, consider second-kind integral equations of the form

$$(I-\mathbf{K})x = y, \tag{1.1}$$

where $\mathbf{K}$ is a bounded linear operator on X^+, the Banach space of bounded continuous functions on $\mathbf{R}^+ = [0,\infty)$ with the supremum norm, and is given by

$$(\mathbf{K}x)(s) = \int_0^\infty K(s,t)x(t)\,dt, \quad s \in \mathbf{R}^+,\ x \in X^+. \tag{1.2}$$

Consider the case where the kernel of the half-line operator is of the form

$$K(s,t) = \kappa(s-t) + K_1(s,t), \tag{1.3}$$

where $\kappa \in L_1(\mathbf{R})$ and $K_1(s,t)$ is a "short ranged" kernel satisfying

$$\sup_{s\in\mathbf{R}^+} \int_0^\infty |K_1(s,t)|\,dt < \infty, \tag{1.4a}$$

$$\lim_{s'\to s} \int_0^\infty |K_1(s',t) - K_1(s,t)|\,dt = 0 \quad \text{uniformly for } s \in \mathbf{R}^+, \tag{1.4b}$$