HOLDER ESTIMATES FOR THE CAUCHY INTEGRAL ON A LIPSCHITZ CONTOUR

WILLIAM MCLEAN

ABSTRACT. Some classical results of J. Plemelj and I. Privalov, concerning Hölder continuity of the Cauchy integral, are generalised by relaxing the smoothness assumptions on the contour of integration.

Introduction. In order to develop the theory of one-dimensional singular integral equations, it is first necessary to establish certain basic properties of the Cauchy integral. Chief among these are the Plemelj-Sokhotski formulae, and estimates in $L_p$ or Hölder norms. Modern treatments of the theory can be found in the books of Gohberg and Krupnik [3], Prößdorff [11] and Mikhlin and Prößdorff [9]; the standard classical text is of course Muskhelishvili [10]. These authors all assume that the contour of integration $\Gamma$ is reasonably smooth - more precisely, $\Gamma$ must consist of a finite number of non-intersecting Lyapunov curves, and must not possess any cusps. (A curve satisfies the Lyapunov condition if and only if it is locally the graph of a function with a Hölder continuous derivative, see [7, Appendix A]).

The aim of this paper is to show that the basic Hölder estimates remain valid when $\Gamma$ is assumed only to be a Lipschitz contour. It is then a relatively straightforward matter to generalise most of the remaining theory of one-dimensional singular integral equations with Hölder continuous coefficients - the details have been worked out in [8]. Happily, the Hölder estimates can be proved using only elementary methods, in stark contrast to the $L_p$ theory for Lipschitz contours, which relies on the Coifman-McIntosh-Meyer Theorem [1].

The paper is organized as follows. In §1, some properties of Lipschitz contours are discussed, along with the important idea of a nontangential limit. The fundamental result, that the Cauchy integral determines a bounded linear operator on spaces of Hölder continuous functions, is proved in §2. Finally, in §3 we establish the Plemelj-Sokhotski formulae.

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